re(Solve)

MATHS BY INQUIRY

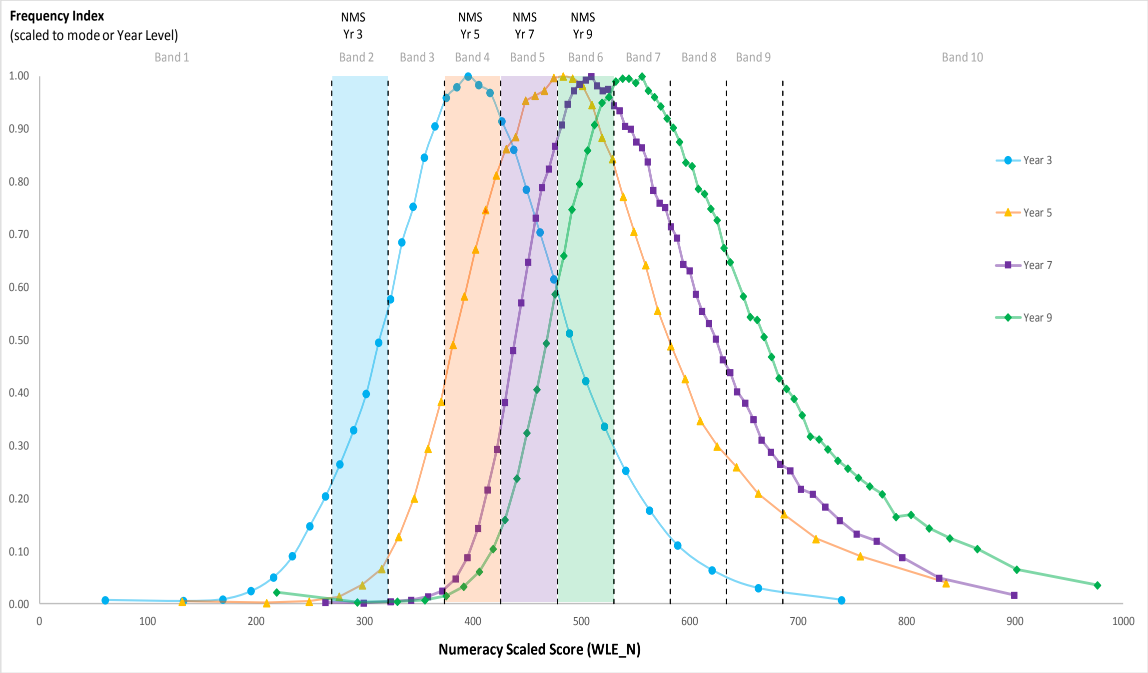
Australian Academy of Science

aamt

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

MODULE 3

Including all students in mathematics learning experiences



This could be the range of students in your class…

and you have to teach all of them…

Module 3 Overview

Part 1 The elements of the reSolve project

Part 2 Considering the nature of difference

Part 3 Some strategies you can use

- Building learning on a common experience
- Low floor, high ceiling tasks
- Enabling and extending prompts

Part 4 Applying the strategies to other tasks

Part 5 Using these approaches in planning and teaching

Part 1: The reSolve Project

Click here if you have already done a PD Module

Overview of the reSolve: Mathematics by Inquiry Project

Classroom Resources

Exemplary classroom resources at every level from Foundation to Year 10 that embody a spirit of inquiry and enact the reSolve Protocol.

Classroom Resources

FOUNDATION - STRUCTURE OF NUMBER: HANDFULS (TRIAL)

ACMNA001; ACMNA002; ACMNA003

YEAR 5 MULTIPLICATION: RESOLVE BAKERY

ACMNA100

YEAR 8 CIRCUMFERENCE

ACMMG197

Special Topics

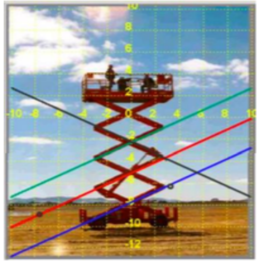
Addressing identified needs or exploring new boundaries.

Special Topics are significant resources that address the needs of 21st century learners. They:

- are substantial units of work and accompanying resources that address major current gaps;
- prioritise the Australian Curriculum proficiencies of reasoning and problem solving;
- provide imaginative opportunities for creatively using new technologies and real world contexts; and
- respond to the results of international assessments that show that solving real problems is a specific area of weakness for Australian students.

Special Topics

Mechanical Linkages and Deductive Geometry	Bring Algebra into the Real World	Modelling Motion	Bar Model Method
Assessing Reasoning	Mathematics and Algorithmic Thinking	Mathematical Modelling	Mathematical Inquiry into Authentic Problems



Professional Learning Modules

Resources to support in-school leaders to address key issues and themes in the teaching and learning of mathematics through collegial professional learning programs.

Professional Learning Modules

PLM 1: reSolve: Mathematics by Inquiry. Introducing the reSolve Protocol and some of the resources	PLM 2: Mathematical purpose and potential	PLM 3: Including all students in mathematics learning experiences	PLM 4: The role of challenging mathematical experiences in activating thinking of all students.
PLM 5: Using student strategies and solutions as part of inquiry learning in mathematics	PLM 6 Creating mathematical inquiries for your students	PLM 7: Activating mathematical thinking then consolidating learning	PLM 8: Leading the incorporation of inquiry approaches into mathematics teaching repertoires: A workshop for current and potential leaders

reSolve Champions

A community of more than 290 committed leaders across the country, who will use reSolve resources and approaches in professional learning programs they lead during and after the development phase of reSolve (finishes mid-2018).

The reSolve Protocol is the driver for all that the project is about, and does.

In order to foster a spirit of inquiry in all mathematics teaching and learning:

- reSolve mathematics is purposeful
- reSolve tasks are challenging yet accessible
- reSolve classrooms have a knowledge-building culture

In particular, this module addresses these statements from the Protocol

Providing opportunity for all students irrespective of background and experience.

Providing prompts and activities meeting a range of student capabilities, from those needing assistance to those ready for further challenge.

This module addresses (in particular) the following AITSL Teaching Standards:

- Standard 1 – Know students and how they learn
  - 1.5 Differentiate teaching to meet the specific learning needs of students across the full range of abilities
  - 1.6 Strategies to support full participation of students with disability
- Standard 4 – Create and maintain supportive and safe learning environments
  - 4.1 Support student participation

Part 2: The nature of difference

The nature of difference

- ACARA says that in any class there is a 5 year gap.
- Cockcroft (UK) reported a 7 year gap.
- While we know that there are factors contributing to difference (experience, Indigenous, SES, geography, gender, race, ethnicity), even within these subgroups there is diversity.
- Differences are not just in achievement, but aspirations, expectations, resilience, mindsets, confidence, and satisfaction.

We are interested in …

- Purposeful learning for everyone

    - *over the year all students should improve by at least 12 months*

- Not merely replicating student prior experience but extending it

    - *not "more of the same"*

- Efficient planning

    - *not preparing different lessons for different sub groups*

Some assumptions that underlie planning to include all students:

Warning: these assumptions may be confronting for some viewers.

Some assumptions that underlie planning to include all students:

All students can learn and have a right to the full range of curriculum experiences

- If I prevent students from having access to the full curriculum,
I am perpetuating inequity and discrimination

Some assumptions that underlie planning to include all students:

Students learn in different ways and at different rates and teaching needs to accommodate these differences

- *Our professional experience as teachers confirms this*
- *Our professional expertise enables this in our classrooms*

Some assumptions that underlie planning to include all students:

The building of relationships, learning to communicate mathematically and learning from each other is easier when the whole class can discuss their experience together

- Quality education has always been a product of community

Some assumptions that underlie planning to include all students:

It is in whole class discussions that students can see alternate approaches and build connections between different ways of seeing concepts

- Our diversity is our strength. Our learning is enriched by finding multiple solutions and strategies

Some assumptions that underlie planning to include all students:

We want to minimise the effects of self-fulfilling prophecy by including all students in whole class teaching

- *It is easier to destroy self-concept than it is to build it*

What do you see as the implications of such statements for planning, teaching and assessment?

Part 3: Three strategies for planning to address difference

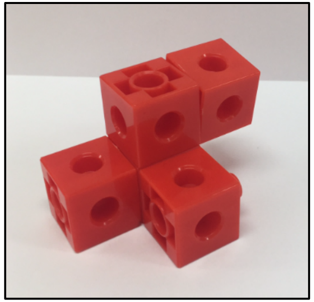
Strategy 1. Providing experiences to give all students access to the subsequent tasks (without reducing the challenge of those tasks).

Strategy 2. Posing tasks with “low floors” and “high ceilings” so that all students are working on the same task but in different ways.

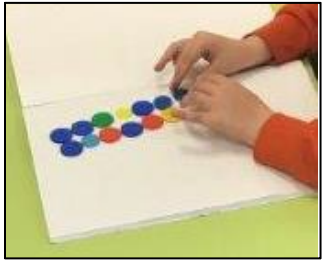
Strategy 3. Preparing additional prompts that can support students experiencing difficulty and extend those who are ready.

Note that all three strategies can be applied separately or together.

Examples of what the three strategies look like



Building Symmetry
Year 3



Handfuls
Foundation

Mathematical Mind Reading
Year 9

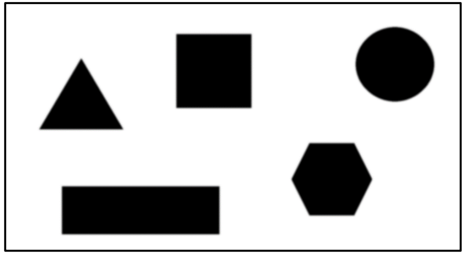
Chinese Bamboo
Year 10

Part 4: Applying these strategies to other tasks

What were they again?

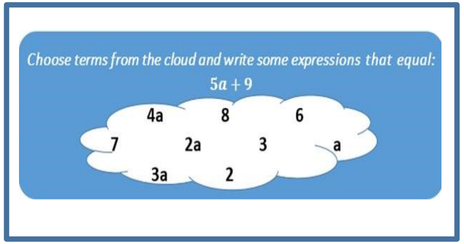
1. Providing initial experiences that give access for all students
2. Low floor, high ceiling
3. Prompts
   a) Enabling – review the representation, number of steps, size of numbers
   b) Extending – increase complexity and size of numbers, find multiple solutions, identify all possibilities, make a generalisation or rule

Choose a task to discuss.



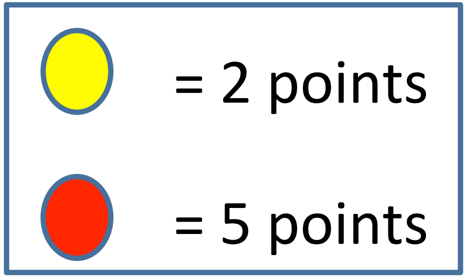
Shadows

Year 1



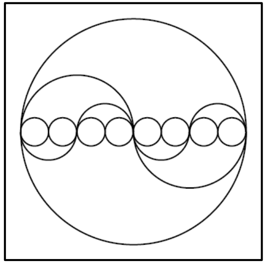
Algebra Cloud

Year 7



Counter Toss

Year 3



Jamos the Jeweller

Year 8

Part 5: Applying these approaches in planning and teaching

What explicit language related to inclusion should we use …
What explicit language related to inclusion should we avoid …

… when communicating with students?

… when communicating with each other?

… when communicating with parents?

How will our approach to making mathematics accessible for all students affect our planning …

… as we meet together?

… in our documentation?

… in our assessment and reporting?

… in our resourcing?

How will we communicate our approach to making mathematics accessible for all?

… in each classroom?

… across the school?

… within the community?

Putting it into practice

- These approaches are ways of thinking that can apply to all tasks and all interactions with students.
- If we created guidelines for ourselves to encourage students to keep working on a task without taking away their opportunity to make their own meaning, what would those guidelines be?

What specific actions do you (as an individual) commit to as a result of participation in this professional learning module?

Part 6: Evaluation

- Please provide us with feedback on this session by typing the URL below, or by using the QR code with your mobile device

https://www.surveymonkey.com/r/PD3evaluation

- It will only take a few minutes.

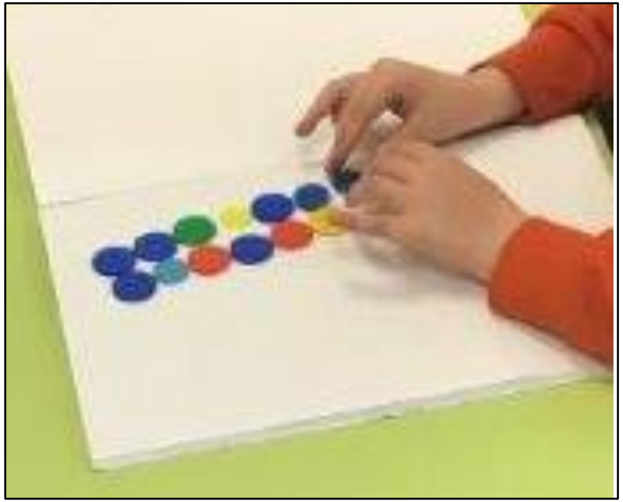
Strategy 1: Providing experiences that give all students access to the subsequent tasks

How does this “experience” increase the chances of **all** students engaging with the subsequent task?

The *experience* of Handfuls …

Present students with materials and have them take a handful.

Ask students to estimate how many items they have in their handful.



… and the subsequent Handfuls *task* …

Carefully count your collection.

Organise your collection in a way that someone else could quickly see how many items there are in your collection.

Explain why your collection might be easy to count.

Strategy 2: Posing tasks with "low floors" and "high ceilings" so that all students are working on the same task even if in different ways.

In the Handfuls task

A response that demonstrates the **low floor** of this task might be when students …

- count their collection correctly even if not arranged in an organised way

A response that demonstrates the **high ceiling** of this task might be when students …

- choose from several different arrangements the one that most easily enables the objects to be counted

Strategy 3: Preparing additional prompts that can
a) support students experiencing difficulty
b) extend those who are ready

Supporting students experiencing difficulty – the power of "enabling prompts"

- "Telling" students experiencing difficulty what to do runs the risk of entrenching their sense of helplessness.
- After completing the prompt, the intention is for students to proceed with the original task.
- As a result, those students can feel part of the class community …
- … as well as learning self-help strategies.

An enabling prompt involves reducing the complexity of the original task by changing …

- The nature of the representation
- The number of steps
- The size of the numbers

Handfuls - enabling prompts

Changing the size of the numbers – Suggest that students take smaller handfuls.

Changing the nature of the representation – Provide resources such as ten frames to help organise their collection.

Extending students who are ready – the use of "extending prompts"

- Extending prompts **explore the potential more deeply**
- Extending prompts:
  - Increase complexity and size of numbers
  - Find multiple solutions
  - Identify all possibilities
  - Make a generalisation or rule
- Extending prompts **do not** change contexts – the prompt still meets the purpose of the lesson

Handfuls – extending prompts

Find multiple solutions: Suggest that students examine and describe similarities or differences between arrangements and discuss the advantages of one over another.

Back to the task selection slide

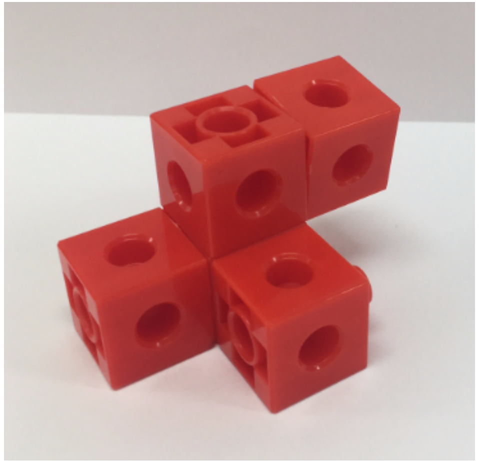
Jump to Part 4

Strategy 1: Providing experiences that give all students access to the subsequent tasks

How does this "experience" increase the chances of **all** students engaging with the subsequent task?

The Building Symmetry *experience* …

Briefly show this structure and have students recreate it from memory.



…and the subsequent Building Symmetry *task*…

How many symmetrical shapes can you make by moving one block?

How many symmetrical shapes can you make by adding one block?

Strategy 2: Posing tasks with “low floors” and “high ceilings” so that all students are working on the same task even if in different ways.

In the Building Symmetry task

A response that demonstrates the **low floor** of this task might be when students …

- give at least one correct design

A response that demonstrates the **high ceiling** of this task might be when students …

- visualise potential solutions and use mental rotation to recognise that some solutions are equivalent

Strategy 3: Preparing additional prompts that can
a) support students experiencing difficulty.
b) extend those who are ready.

Supporting students experiencing difficulty – the power of "enabling prompts"

- "Telling" students experiencing difficulty what to do runs the risk of entrenching their sense of helplessness.
- After completing the prompt, the intention is for students to proceed with the original task.
- As a result, those students can feel part of the class community …
- … as well as learning self-help strategies.

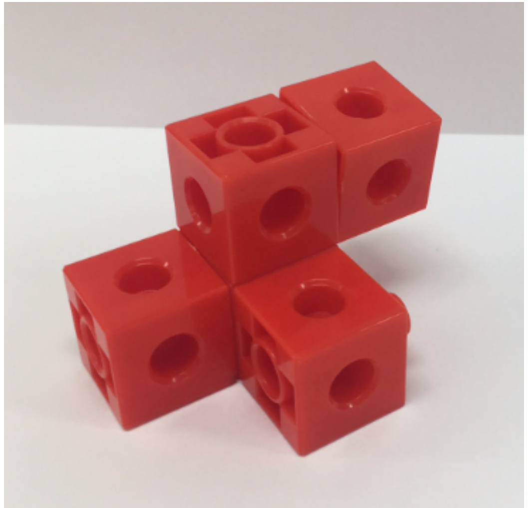
An enabling prompt involves reducing the complexity of the original task by changing …

- The nature of the representation
- The number of steps
- The size of the numbers

Note that sometimes it may be appropriate to use the enabling prompt with the whole class if hardly any students can proceed with the original task

Building Symmetry - enabling prompts

- **Reducing the number of steps** – "Remove one block to make your model symmetrical".
- "Look at the new symmetrical model you have made. Can you add a block to your model and still keep it symmetrical?"



Extending students who are ready – the use of "extending prompts"

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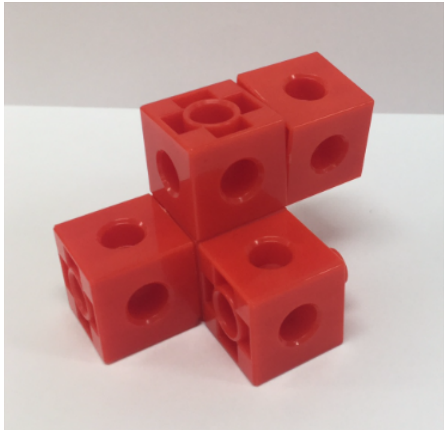
  - Identify all possibilities
  - Make a generalisation or rule
- Extending prompts **do not** change contexts – the prompt still meets the purpose of the lesson

Building Symmetry – extending prompts

Make a generalisation or rule:

"Move a block to make a symmetrical object. Now add one block to keep it symmetrical. In how many places can you add the block?"



Back to the task selection slide

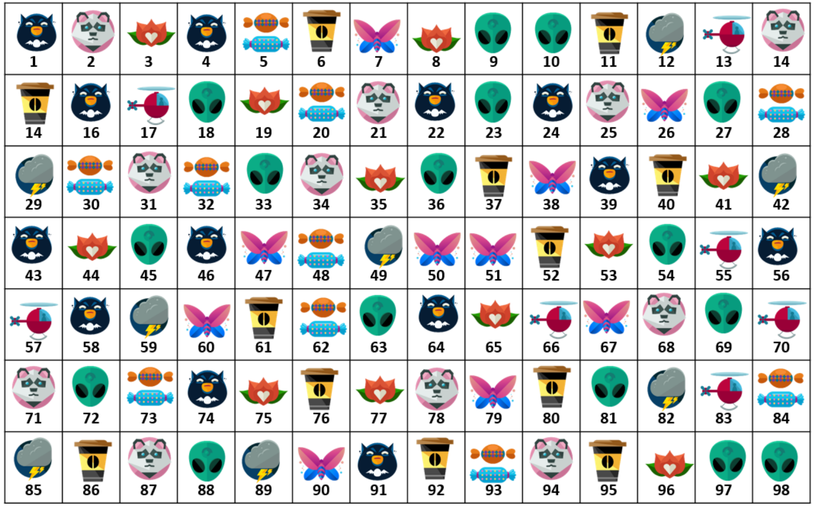
Jump to Part 4

Strategy 1: Providing experiences that give all students access to the subsequent tasks

How does this “experience” increase the chances of **all** students engaging with the subsequent task?

The Mathematical Mind Reading *experience* …

- Write down a 2-digit number whose digits are different
- Reverse the digits
- Subtract the smaller number from the larger number
- Look for the icon that is next to your answer



…and the subsequent Mathematical Mind Reading *task*…

How does this work?

What pattern do the solutions follow?

How can we represent this process algebraically?

Strategy 2: Posing tasks with “low floors” and “high ceilings” so that all students are working on the same task even if in different ways.

Mathematical Mind Reading

A response that demonstrates the **low floor** of this task might be when students …

- recognise that all the answers are multiples of 9

A response that demonstrates the **high ceiling** of this task might be when students …

- explain how the mind reading trick works using algebraic representation

Strategy 3: Preparing additional prompts that can
a) support students experiencing difficulty
b) extend those who are ready

Supporting students experiencing difficulty – the power of "enabling prompts"

- "Telling" students experiencing difficulty what to do runs the risk of entrenching their sense of helplessness.
- After completing the prompt, the intention is for students to proceed with the original task.
- As a result, those students can feel part of the class community …
- … as well as learning self-help strategies.

An enabling prompt involves reducing the complexity of the original task by changing …

- The nature of the representation
- The number of steps
- The size of the numbers

Mathematical Mind Reading – enabling prompts

Changing the nature of the representation – "Look at the animated graphic to visualise how 2-digit numbers can be represented algebraically"

Extending students who are ready – the use of "extending prompts"

- Extending prompts **explore the potential more deeply**
- Extending prompts:
  - Increase complexity and size of numbers
  - Find multiple solutions
  - Identify all possibilities
  - Make a generalisation or rule
- Extending prompts **do not** change contexts – the prompt still meets the purpose of the lesson

Mathematical Mind Reading – extending prompts

Increase complexity and size of numbers: "Can you use algebra to predict what will happen if we do the same process with a 3-digit number?"

Back to the task selection slide

Jump to Part 4

Strategy 1: Providing experiences that give all students access to the subsequent tasks

How does this “experience” increase the chances of **all** students engaging with the subsequent task?

The Chinese Bamboo *experience* …

Take a piece of “bamboo”. Bend it so that the top touches the ground. Where does the bend need to be for it to touch the ground? Where does it touch the ground if the bend is near the base of the bamboo?


詳解九章算法

折抵地爲弦以句及股弦并求股故先令句自乘見矩
冪令如高而一凡爲高一丈爲股弦并之以除此冪得
差所得以減竹高而半其餘卽折者之高也此率與係
索之類更相返覆也亦可如上術令高自乘爲股弦并
冪去本自乘爲矩冪減之餘爲實倍高爲法則得折之
高數也

股弦和與勾求股法曰勾自乘爲實變股弦較乘股弦
和如股弦和而一正除得股弦較以減股弦和餘二段

…and the subsequent Chinese Bamboo *task*…

Given the total height of the bamboo and the horizontal distance from the base of the bamboo and the top, calculate the height to the break.

Compare to the solution given on a traditional Chinese manuscript.

Strategy 2: Posing tasks with "low floors" and "high ceilings" so that all students are working on the same task even if in different ways.

In the Chinese Bamboo task

A response that demonstrates the **low floor** of this task might be when students …

- find a solution using trial and error by choosing a height for the break and using Pythagoras to find the horizontal distance.

A response that demonstrates the **high ceiling** of this task might be when students …

- find the solution using Pythagoras and algebra and see that it is the same as the Chinese solution.

Strategy 3: Preparing additional prompts that can
a) support students experiencing difficulty
b) extend those who are ready

Supporting students experiencing difficulty – the power of "enabling prompts"

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- … as well as learning self-help strategies.

An enabling prompt involves reducing the complexity of the original task by changing …

- The nature of the representation
- The number of steps
- The size of the numbers

Chinese Bamboo – enabling prompts

Changing the nature of the representation – Give students a numerical value for the height of the break and ask for the length of the hypotenuse. Use this to ask them to express the length of the hypotenuse in terms of the height of the break.

Extending students who are ready – the use of "extending prompts"

- Extending prompts **explore the potential more deeply**
- Extending prompts:
  - Increase complexity and size of numbers
  - Find multiple solutions
  - Identify all possibilities
  - Make a generalisation or rule
- Extending prompts **do not** change contexts – the prompt still meets the purpose of the lesson

Chinese Bamboo – extending prompts

Make a generalisation or rule

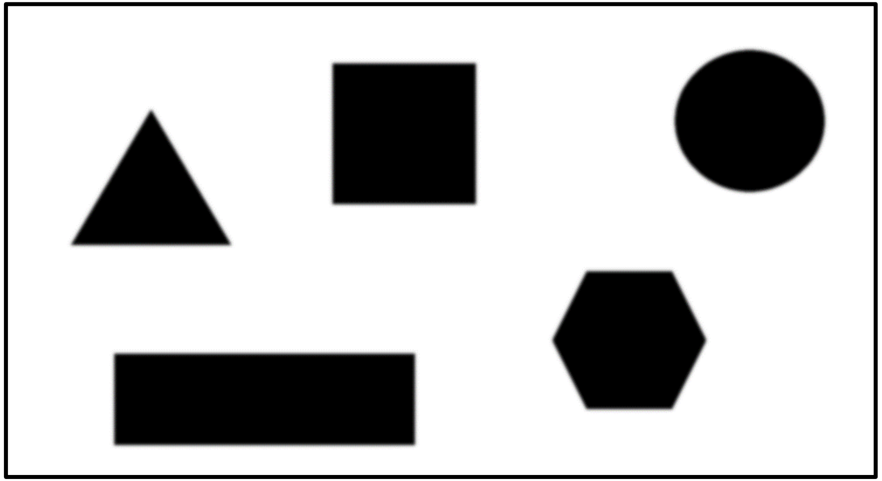
Show the equivalence between the general solution for any length bamboo found using Pythagoras and that given in the Chinese text.

Back to the task selection slide

Jump to Part 4

Shadows

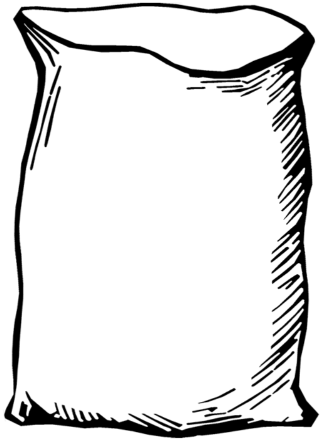
What objects might have made these shadows?



Back to the task selection slide

Jump to Part 5

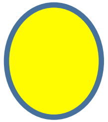
Counter Toss

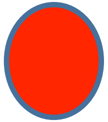


In this bag, I have red counters and yellow counters.

I take out 4 counters.

What are my possible scores?

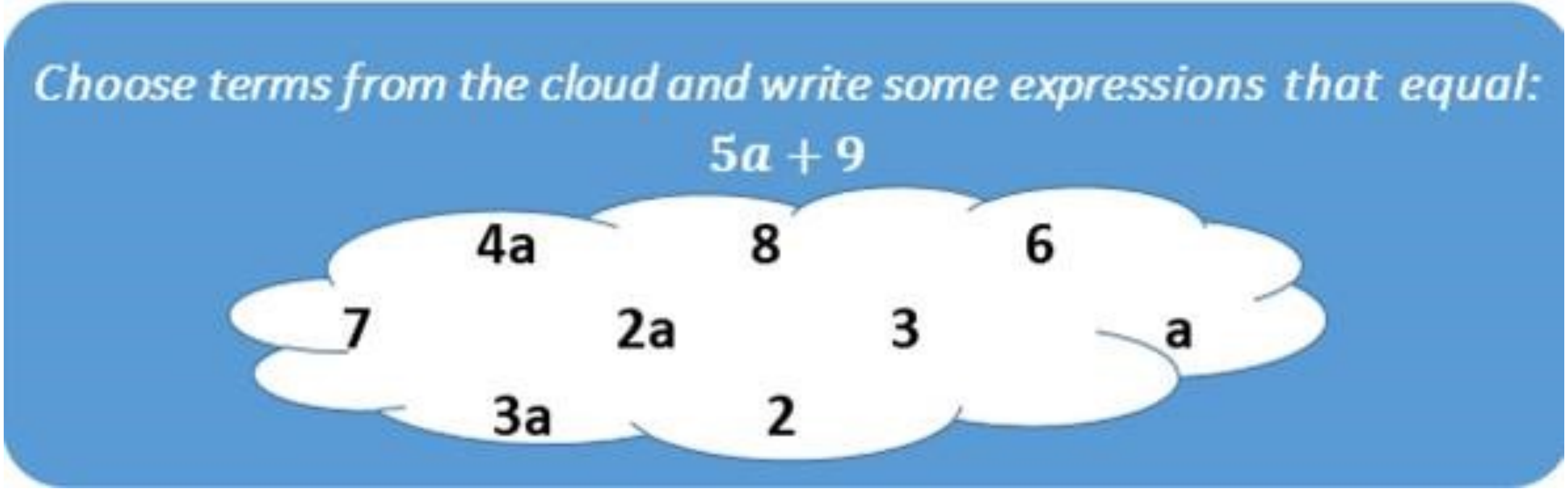
 = 2 points

 = 5 points

Back to the task selection slide

Jump to Part 5

Algebra Cloud



Back to the task selection slide

Jump to Part 5

Jamos the Jeweller

Jamos the Jeweller charges clients based on the amount of precious metal wire used in their jewellery designs.

A customer has provided the following design brief for Jamos to create a new design. Can you help?

> *A symmetrical design, with at least three different sized circles and with the metal costing between $600 and $650*

Jamos charges clients $25 per linear centimetre of wire used.

Back to the task selection slide

Jump to Part 5

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